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CALCULATIONS OF MOLTIPOLE MIXING RATIOS FOR GAMMA TRANSITIONS OF ^{172}Yb POPULATED FROM $^{172}yb(n,n^{'}\gamma)$ REACTION USING a_2- RATIO, CONSTANT STATISTICAL TENSOR (CST) AND LEAST SQUARES FITTING (LSF) METHODS

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ABSTRACT

The δ - mixing ratios of γ-transitions from levels in ^{172}Yb populated in the$^{172}yb(n,n'\gamma)$reactions are calculated in present work using a_2- ratio, constant statisticalTensor (CST) and least squares fitting (LSF) methods

The results obtained are in general, in good agreement or consistent, within the associated uncertainties, with these reported in Ref.[9],the discrepancies that occurs are due to inaccuracy existing in the experimental data

The results obtained in the present work confirm the CST –method for mixed transitions better than that for pure transition because this method depends only on the experimental data where the second method depends on the pure or those considered to be pure γ-transitions, the same results occur in LSF – method

KEYWORDS: $\delta -$ Mixing Ratios, $\gamma -$ Transitions, a_2- Ratio, Constant Statistical Tensor (CST), Least Squares Fitting (LSF) Methods,$^{172}yb(n,n'\gamma)$ reaction .

INTRODUCTION

- Gamma-Ray Transition

The emission of an energetic quantum of electromagnetic energy is called γ-ray for γ-transition from an initial state of spin J_i and parity π_i to a final state of spin J_f and parity π_f the transition by emission of single 2^L pole quantum is possible if :

$$\left|J_i - J_f\right| \leq L \leq J_i + J_f \quad L \neq 0 \tag{1}$$

Where:

L: is the angular momentum of the γ-transition in such transitions, the parity change of electric radiation, EL, is given by: $\pi_i . \pi_f = (-1)^L$ (2)

And for magnetic radiation, ML, by:

$$\pi_I \ \pi_f = (-1)^{L+1} \tag{3}$$

If the initial and final parities are equal, then $M1, E2, M3, E4, M5$, etc .will conserve parity .If the parities of the initial and final state are different, then $E1, M2, E3, M4, E5$,etc.are possible.

The wavelength of the emitted radiation, λ, is much greater than the dimension, R, of the radiation system. The intensity of radiation from each successively higher multipoles is therefore, less by a factor $(R/\lambda)^{2L}$ which means that$E1 >$

$E2 > E3 \ldots . And\ M1 > M2 > M3$

The parity change rule coupled with this factor indicates that, for transitions between states of well-defined parity, the only multipole mixing is expected to be the type ML and $\mathrm{E(L+1)}$ or E1 and $M(L+1)$.However, the magnetic transitions are expected to be approximately$(V/C)^2$ times slower than the electric transitions of the same multipole order (V is the speed and C is the speed of the light[1]. Assuming $R = 1.2\ A^{\frac{1}{3}}\, F$, where A is the mass number of the radiating system (excited nucleus), then for the same L,[2,3].

$$\lambda(ML) \cong 0.3\ A^{-\frac{2}{3}} \lambda(EL) \tag{4}$$

This indicates that magnetic multipole radiation is less probable than electric multipole radiation of the same order, that is E1> M1 and E2> M2

The first type multipole mixing is therefore, much more likely than the second one .This is sometimes called parity favored mixing .In such a transition, if δ is the multipole mixing ratio, then [1]

$$\delta^2 = \Gamma\,(L_2)\,/\,\Gamma\,(L_1) \tag{5}$$

Where

$L_2 = L_1 + 1$ And $\Gamma\,(L_1) + \Gamma(L_2) = \mathit{\Gamma}_\gamma$ (total gamma width)given by:

$$\mathit{\Gamma}_\gamma . \tau = \hbar = 0.6582 \times 10^{-15} \mathrm{eV.s} \text{ (Dirac constant)} \tag{6}$$

Where

ƭ = is the mean life of the initial state, ħ =h /2π and h: is plank constant. In the case of transitions where either $J_i$or J_f is zero L is uniquely defined and hence only electric or only magnetic transition are allowed by the parity change rule. This means that multipole mixing is very much unlikely in this case [1].

If both J_i and J_f are zero, then radiative transitions by single photon emission are strictly for bidden since

L≠ 0 for gamma transition

- a_2-Ratio Method

This method depends only on the experimental a_2 –coefficient determined for at least two γ-transitions from the same initial state, one of with is a pure transition such as (1-0) or (2-0) transitions or might be considered a pure $E1$ transition such as (2^- - 2^+), (3^- - 2^+),(3^- - 4^+), etc. or a pure $E2$ transition such as 4-2,5-3,6-4,…etc.

In such cases, the statistical tensor $\rho_2(J_i)$ which is related to thea$_2$ – coefficient would be the same for the two transitions and hence the δ –values for the other transition can be calculated from the ratio of the experimental a_2-coefficient of this transition to that of the a pure transition. This method has been successfully applied by Youhana et .al [4,5] and by Youhana [6].

Constant Statistical Tensor (CST) Method

The magnetic sub state population parameters of levels excited in ^{92,94}Zr(n,n'γ) reactions have been calculated by Ameen [7] using the computer program POP which is are duced and modified version of the computer code CINDY[8]. It

was found that the population parameters of levels in both isotopes was almost constant for levels with the same spin value for both parties .It was therefore concluded that the population parameters of levels with the same spin value depend neither upon the energy of the level nor upon its parity.

Depending upon this fact, Youhana [10, 11] has shown that the statistical tensor, which is related to the population parameters should also be constant for levels with the same spin value and as a result, the constant statistical tensor, which is related to the population parameters should also be constant for levels with the same spin value and as a result, the constant statistical tensor (CST) method was suggested as a tool to calculate the multipole mixing ratios, δ, of γ- transitions. This method was then applied for the first time by Youhana [10,11] to calculate the δ-values γ-transition from levels in ^{90,92,94}Zr and ^{150}Nd excited in the (n,n'γ) reactions. In these studies, Youhana has not confirmed the validity of this method as a tool, as good as the computer cod CINDY [8], for calculating the δ-values of gamma transitions only but also its capability of predicting any in accuracy existing in the experimental data .In addition the CST method depends upon the experimental data and dos not depend upon any nuclear model .The method is also rather simple and a modern electronic calculator is quite enough to perform all the necessary calculations .

- Least Square Fitting (LSF) Method

Levels with certain J_i values might have no pure γ-transitions or transitions which is considered to be pure. The statistical tensor $\rho_2\,(J_i)$ for such levels cannot be calculated as mentioned and hence the δ-values of mixed transitions from such levels cannot be determined by the CST –method.

It also happens that a level with certain $J_i$value has only one pure or considered to be pure γ-transition whose a_2- coefficient is not accurately measured, in which case, the $\rho_2\,(J_i)$ calculated for that level shall not be accurate also. The LSF-method was, therefore, suggested to estimate $\rho_2\,(J_i)$ for all $J_i$values

Theoretical Concept

- Gamma Ray Angular Distribution

Angular distribution is defined as the distribution in angles relative to an experimentally specified direction, of the intensity of photons or particles usually resulting from a nuclear reaction [12]

The angular distribution of gamma transition from initial state of spin J_i (magnetic quantum number) to a final state of spin J_f (magnetic quantum number m_f) can be expressed by [13].

$$W(\Theta) = \sum_k A_k P_k \,\mathrm{CoS}\,\Theta$$

$$W(\Theta) = \sum_k P_k(J_k) F_k(J_i J_f \delta) P_k \cos\Theta \quad (7)$$

Where

A_k= is the angular distribution coefficient.

Θ = is the angle between the direction of the γ- rays and the axis of alignment (beam direction).

$\mathbf{P_k cos\,\theta}$ **:** is the legender polynomial .

$\boldsymbol{\rho_k\,(J_i)}$ **:** are statistical tensor which describe the alignment of the initial state.

$\boldsymbol{F_k\ (J_iJ_f\delta\ )}$**:**Coefficients which contain the information on angular momentum changes and the multipole mixing ratios.

In general the $F_k(J_iJ_f\,\delta)$ coefficients are given by [14]

$$F_k(J_i\ J_f\,\delta)=\frac{F_k\,(J_fL_1L_1J_i)+2\delta\,F_k\,(J_f\,L_1L_2J_i)+\delta^2\,F_2\,(\,J_f\,L_2L_2J_i\,)}{(1+\delta^2)} \tag{8}$$

Where: $L_2= L_1+1$,$\boldsymbol{\delta}$: is the mixing ratio, $F_k(J_i\ J_f\,\delta)$ =coefficients are given by [14].

$$F_k(J_fL_1L_2J_i)=(-1)^{J_i-J_f-1}[(2L_1+1)(2L_2+1)(2J_i+1)]^{1/2}(L_1 1\ L_2-1\,|\,K0).W(J_iJ_i\,L_1L_2\ K\,J_f\,) \tag{9}$$

Where

$(L_1 1\ L_2-1\,|\,K0)$:are Clabsch –Gordon Coefficients and $W(J_iJ_i\,L_1L_2\ K\,J_f\,)$: are Racah Coefficients

The triangular condition on the Racah Coefficients limit k to [13]

$$0\leq k\leq \min(2L_{1,}2L_2,2J_i) \tag{10}$$

For $k=0$, $F_0(J_f\ L_1\ \ L_2\ \ J_i)=\delta L_1L_2=\begin{cases}1\ if\ L_1=L_2\\0\ if\ L_1\neq L_2\end{cases}$ **(11)**

The statistical tensor,$\boldsymbol{\rho_k}\ (J_i\,)$ are given by a weighted sum over the population parameter, $P(m_i\,)$ of the $(2J_i+1)$ magnetic substrates associated with$(J_i\,)$ [13].

$$\rho_k\,(J_i\,)=\sum_{\substack{m_i=0\\ or\ m_i=1/2}}^{Ji}\rho_k\,(J_i\,,m_i)P(m_i) \tag{12}$$

With the normalization

$$\sum_{m_i=-J_i}^{J_i}P(m_i)=1 \tag{13}$$

For an aligned and un polarized initial a state,

$$P(m_i\,)=,P(-m_i\,) \tag{14}$$

So that , $P(m_i\,)$ values are in the range,

$$0\leq,P(m_i\,)\leq 1/2\,(1+\delta_{min}) \tag{15}$$

And hence,

$$\rho_k\,(J_i\,,m_i)=(2-\ \delta_{min})\frac{(\,J_im_iJ_i-m_i\,|\,K0)}{(J_i\,m_iJ_i-m_i\,|\,00)} \tag{16}$$

The Clesch – Gordan *coefficients,* $(\,J_im_iJ_i-m_i\,|\,K0)$, are zero for odd values of k and hence eq.(7) contains only even values of k .

From eq. (16) it is clear that

$$\rho_k(J_i, m_i) = (2 - \delta_{min}) \quad (17)$$

And hence $\rho_0(J_i)=1$

From eq. (8) its clear that $\rho_0(J_i) = F_0(J_i J_f) = 1$ and $a_k = A_k / A_0$, then

$$a_k = \rho_k(J_i) \frac{F_k(J_f L_1 L_1 J_i) + 2\delta F_k(J_f L_1 L_2 J_i) + \delta^2 F_k(J_f L_2 L_2 J_i)}{(1+\delta^2)}. \quad (18)$$

- **a_2 –RatioMethod**

This method depends only on the experimental a_2-coefficients obtained for at least two γ-transitions from the same level one of which is pure transition or may be considered as a pure transition

The a_2-coefficient of the pure transition from a certain initial level is given by:

$$a_2(J_i - J_{f_1}) = \rho_2(J_i) F_2(J_{f_1} L_1 L_1 L_1 J_i) \quad (19)$$

The mixed transition from the same level $a_2(J_i - J_{f_2})$ is given by:

$$a_2(J_i - J_{f_2}) = \rho_2(J_i) \frac{F_2(J_{f_2} L_1 L_1 J_i) + 2\delta F_2(J_{f_2} L_1 L_2 J_i) + \delta^2 F_2(J_{f_2} L_2 L_2 J_i)}{(1+\delta^2)} \quad (20)$$

Where

$\rho_2(J_i)$ is the same for both transitions, and hence

$$\frac{a_2(J_i - J_{f_2})}{a_2(J_i - J_{f_1})} = \frac{F_2(J_{f_2} L_1 L_2 J_i) + 2\delta F_2(J_{f_2} L_1 L_2 J_i) + \delta^2 F_2(J_{f_2} L_2 L_2 J_i)}{F_2(J_{f_1} L_1 L_1 J_i)(1+\delta^2)} \quad (21)$$

- **Constant Statistical Tensor (CST) Method**

This method depends on the fact that in a certain nucleus, the magnetic sub states population parameters, $P(m_i)$, of levels with the same spin value depend neither upon the energy of the level nor upon its parity[8]

The statistical tensor coefficients $\rho_k(J_i, m_i)$ are also constant for the same J_i values. Then according to eq.(11) the statistical tensor $\rho k(J_i)$, would also be constant for levels with the same J_i values .Taking this fact into consideration the experimental value of the angular distribution coefficients, a_2 obtained for certain and well known γ-transitions such as $|J_i - J_f| = 2$ with $\pi_i.\pi_f = +1$ **or** $|J_i - J_f| = 0$ or 1 with $\pi_i.\pi_f = -1$, can be used to calculate the statistical tensor $\rho_2(J_i)$ for initial levels for such transition using eq. (17),and putting δ=0 sine such transitions may be considered to be pure $E2$ or pure $E1$ transitions.

The $\rho_2(J_i)$ values thus obtained may then be used to calculate the δ-values for other transitions such as (1^+-2^+),(2^+- 2^+),(3^+- 2^+),(3^+- 3^+),(3^+- 4^+),(4^+- 4^+). Using eq.. (17)

- **Least Squares Fitting LSF Method**

The $\rho_2(J_i)$ values are calculated for levels with different J_i values are computer fitted to polynomial series, of the from

$$\rho_2(J_i)= \sum_{i=0}^{i=n} A_i J_i \tag{22}$$

With n=1,2,3,and 4using the least squares fitting program (LSF) that was written to determine the A_i parameters for all n-values and the χ^2-values for each n. The set with minimum χ^2 is then used to calculate the $\rho_2(J_i)$values for all J_i-values.

DATA REDUCTION ANALYSIS

1- The a_2-Ratio Method

In this method, the levels of the product nuclei that have at least two transitions whose angular distributions have been measured and one of which is pure transition or can be considered as a pure γ-transition are taken into consideration

The δ-mixing ratios of other transition can then be calculated using eq. (20) and the values of the F_2–coefficients presented in ref [17]

In this equation, $a_2\,(J_i - J_f)$ represent the experimental a_2–coefficient reported for the pure transition. The gamma-ray angular momentum L_1 is taken to be

$L_1 = |J_i - J_f|$ And $L_2 = L_1 + 1 L_1 \neq 0$

For the possible γ-transitions, eq. (19) become in the case of the [172]Yb product nuclei as a follows:-

$$\frac{a_2\,(1^- - 2^+)}{a_2\,(1^- - 0^+)^*} = \frac{+0.07071 + 0.94868\,\delta + 0.35355\delta^2}{+0.70711\,(1+\delta^2)} \tag{23}$$

$$\frac{a_2(2^+ - 2^+)}{a_2(2^+ - 0^+)^*} = \frac{-0.41833 - 1.22476\delta + 0.12806\delta^2}{-0.59761\,(1+\delta^2)} \tag{24}$$

$$\frac{a_2(2^+ - 2^+)}{a_2(2^+ - 4^+)} = \frac{-0.41833 - 1.22476\delta + 0.12806\delta^2}{-0.17075(1+\delta^2)} \tag{25}$$

$$\frac{a_2\,(2^+ \text{-} 4^+)}{a_2\,(2^+ \text{-} 2^+)} = \frac{-0.17075 + 1.01014 + 0.44822\delta^2}{-0.41833(1+\delta^2)} \tag{26}$$

$$\frac{a_2\,(3^- - 4^+)}{a_2\,(3^+ - 2^+)} = \frac{+0.14434 + 1.44338\,\delta + 0.30929\,\delta^2}{+\,0.142372\,(1+\delta^2)} \tag{27}$$

$$\frac{a_2\,(3^+ - 2^+)}{a_2\,(3^- - 4^+)} = \frac{+0.34641 - 1.89738\,\delta - 0.12372\,\delta^2}{-0.40764\,(1+\delta^2)} \tag{28}$$

$$\frac{a_2\,(3^- - 2^+)}{a_2\,(3^- - 4^+)} = \frac{+0.34641 - 1.89738\,\delta - 0.12372\,\delta^2}{+0.11554\,(1+\delta^2)} \tag{29}$$

$$\frac{a_2\,(3^- - 4^+)}{a_2\,(3^- - 2^+)} = \frac{+0.14434 + 1.44338\,\delta + 0.30929\,\delta^2}{+0.28912\,(1+\delta^2)} \tag{30}$$

$$\frac{a_2\,(3^- - 2^+)}{a_2\,(3^- - 4^+)} = \frac{+0.34641 + 1.89738\,\delta - 0.12372\,\delta^2}{+0.15878\,(1+\delta^2)} \tag{31}$$

$$\frac{a_2\,(3^- - 4^+)}{a_2\,(3^- - 2^+)} = \frac{+0.14434 + 1.44338\,\delta + 0.30929\,\delta^2}{+0.30829\,(1+\delta^2)} \tag{32}$$

The star (*) in same equations indicates that the parity of the initial state and final state are different, so that, the γ-transitions are pure $E1, E2$ transitions.

2- The Constant Statistical Tensor (CST) Method

According to eq. (17), δ=0 for pure transitions or δ-transitions considered to be pure. The stistical tensor can then be calculated from the following relationship:-

$$\rho_2(J_i) = \frac{a_2(J_i - J_f)}{F_2(J_f L_1 L_1 J_i)} \qquad (33)$$

For all J_i–values of levels having one or more pure gamma transition

In the case of the product nuclei, ^{172}Yb eq. (33) becomes:-

$$\rho_2(1) = \frac{a_2\,(1^- - 0^+)^*}{F_2\,(0111)} = \frac{a_2\,(1^- - 0^+)^*}{+0.70711} \qquad (34)$$

$$\rho_2(1) = \frac{a_2\,(1^- - 2^+)}{F_2\,(2111)} = \frac{a_2\,(1^- - 2^+)^*}{+0.07071} \qquad (35)$$

$$\rho_2(2) = \frac{a_2\,(2^+ - 0^+)^*}{F_2\,(0222)} = \frac{a_2\,(2^+ - 0^+)^*}{-0.59761} \qquad (36)$$

$$\rho_2(2) = \frac{a_2\,(2^+ - 2^+)}{F_2\,(2112)} = \frac{a_2\,(2^+ - 2^+)}{-0.41833} \qquad (37)$$

$$\rho_2(2) = \frac{a_2\,(2^+ - 4^+)}{F_2\,(4222)} = \frac{a_2\,(2^+ - 4^+)}{-0.17075} \qquad (38)$$

The star (*) also indicates that the γ-transitions one considered to be pure.

The statistical tensors $\rho_2(J_i)$, are considered to be constant for all levels with the same J_i values. The values obtained for$\rho_2(J_i)$ can then be used to calculate.

The δ-mixing ratios for all –γ- transitions whose angular distribution have been measured other then (1-0) and

(2-0) transitions, using eq.(17)

Taking into consideration such transitions from levels of ^{172}Ybeq(17)becomes:-

$$a_2\,(1\text{-}2) = \rho_2(1)\frac{0.07071 + 0.94868\,\delta + 0.35355\,\delta^2}{(1+\delta^2)} \qquad (39)$$

$$a_2\,(2\text{-}2) = \rho_2(2)\frac{-0.41833 - 1.22476\,\delta + 0.12806\,\delta^2}{(1+\delta^2)} \qquad (40)$$

$$a_2\,(2\text{-}4) = \rho_2(2)\frac{-0.17075 + 1.01014\,\delta + 0.44822\,\delta^2}{(1+\delta^2)} \qquad (41)$$

$$a_2\,(3\text{-}2) = \rho_2(3)\frac{0.34641 - 1.89738\,\delta - 0.12372\,\delta^2}{(1+\delta^2)} \qquad (42)$$

$$a_2\,(3\text{-}3) = \rho_2(3)\frac{-0.43301 - 0.66026\,\delta + 0.22682\,\delta^2}{(1+\delta^2)} \qquad (43)$$

$$a_2\,(3\text{-}4) = \rho_2(3)\frac{0.14434 + 1.44338\,\delta + 0.30929\,\delta^2}{(1+\delta^2)} \qquad (44)$$

$$a_2\,(4\text{-}4) = \rho_2(4)\frac{-0.43875 - 0.67082\,\delta + 0.26455\,\delta^2}{(1+\delta^2)} \qquad (45)$$

$$a_2\,(5\text{-}3) = \rho_2(5)\frac{-0.42056 - 1.11268\,\delta - 0.36799\,\delta^2}{(1+\delta^2)} \qquad (46)$$

3- Least Squares Fitting (LSF) Method

Levels with certain J_i values might have no pure γ-transition or transition considered to be pure .The statistical tensor $\rho_2(J_i)$for such levels cannot be calculated and hence the δ-values of mixed transition from such levels cannot be determined by the CST method .It also happens that a level with certain J_i-values has only one pure γ-transition or considered to be pure γ-transition whose a_2 –coefficient is not accurately measured in which case, the statistical tensor$\rho_2(J_i)$calculated for that level shall be inaccurate also.The LSF method was therefore, suggested to estimate $\rho_2(J_i)$for all J_i-values .The $\rho_2(J_i)$values calculated for levels with different J_i _values, were computer fitted to a polynomial series of the from :

$$\rho_2(J_i)=\sum_{i=0}^{i=n} A_i J^i \quad (47)$$

with n=1,2,3,and 4, using the least squares fitting program to determine the A_i parameters for all n-values and the χ^2 –values for each n. the set with minimum χ^2 was then used to calculate the $\rho_2(J_i)$values for all J_i –values .The$\rho_2(J_i)$values thus obtained were then used to calculate the δ-values of allγ-transitions whose angular distribution have been measured in same manner.

RESULTS & DISCUSSIONS

- **From Table (1) it is Clear That:**

- There are two γ-transitions from the same initial state J_i to J_f state, this mean that we can applied a_2- ratio method in order to calculate multipole mixing ratios for this γ-transitions
- The a_2- ratios method can be used in two cases which are :
- Where one of the two transitions is pure in this caseδ =0 for the mixed transition
- If the two transitions are not pure then we used $\delta - value$ from Ref[9]for the second transition in order to calculate $\delta - value$ for the first transition and vesa- versa.

  Thus we used the experimental values for δ and a_2 to calculate theorticale $\delta - value$ and comparewith it
- By using the a_2 and $\delta - value$from Ref.[9] for 1093.5 Kev $(3^+ - 2^+)$ from 1172.3 Kevinorder to calculate $\delta - value$for 91.9 Kev$(3^+ - 4^+)$ from the same level we see that the two $\delta - value$.there are disacrepencies with $\delta - value$from Ref[9] but, when we used the second $\delta - value$we find that $\delta - value$ for first transition more accurate, this confirm that $\delta - value$ in Ref[9] is not an accurate for the $(3^+ - 2^+)$ γ-transition.
- The same think are accurse for 1476.8 Kev $(2^+ - 0^+)$ from 1476.8 Kev. For other transitions, the calculated $\delta - value$ using this method are in consisted with published$\delta - value$ in Ref[9].This confirm that the experimental result for this transitions are accurate

- **From Table (2) Represented Weighted Average $\rho_2\ (j_i)$Values**

It is clear that the $\rho_2\ (j_i)$ values which has the same (j_i) value are consistent within the associated errors, this indicate that the experimental value for these transitions are accurate

- **From Table (3) Represented $\delta -$ Values Calculated by CST- Method**

It is clear from this table that the calculated $\delta - value$s in CST- method are ingood agreement with those in Ref[9] for mixed transitions but it is not consisted with $\delta - values$reported in Ref[9] this indicatethat this transitions are

not pure transitions.

- **From Table (4) Represented $\delta -$ Values Calculated in LSF – Method**

It is clear from this table that the calculated $\delta - values$in this method are consistent with those in Ref[9] but it is not consisted for pure transitions .This indicate that this transition are not pure .

Table (1) : Multipole Mixing Ratios of Gamma – Transitions from Energy Levels of ^{172}Yb [9] Calculate by the $a_{2\_}$ -Ratio Method

$E_{i\,(keV)}$	$E_{\gamma\,(keV)}$	J_i^π -J_f^π	a_2 a_4[9]	δ[9]	δ $a_2 -$ Ratio(p.w)
1117.9(2)	1039.3	$2^+ - 2^+$	$+0.183(35)$ $-0.056(40)$	$-0.02(7)$ $+2.3^{+0.5}_{-0.3}$	-($0.01^{+0.11}_{-0.13}$) +($2.3^{+0.9}_{-0.6}$)
	857.4	$2^+ - 4^+$	$+0.067(20)$ $-0.030(23)$	$+0.02(4)$	$+0.02(5)$ $-1.7(2)$
1155.0(2)	1155.1	$1^- - 0^+$	$-0.129(54)$ $+0.016(64)$	$E1$	$E1$
	1076.3	$1^- - 2^+$	$-0.018929)$ $+0.010(33)$	$+0.01(15)$	$+(0.03^{+0.13}_{-0.18})$ $-(3.8^{+6.8}_{-1.7})$
1172.3(2)	1093.6	$3^+ - 2^+$	$-0.089(28)$ $+0.083(35)$	-($7.2^{+1.4}_{-0.9}$)	$-(7.2^{+1.7}_{-1.3})$ $0.11^{+0.02}_{-0.18}$
	9119	$3^+ - 4^+$	$+0.255(42)$ $+0056(47)$	$-1.5(4)$	$-\left(1.5^{+?}_{-1.2}\right)$ $-(0.5^{+1.68}_{-?})$
1221.8(2)	1143.1	$3^- - 2^+$	$-0.193(4)$ $+0.011(48)$	$+0.02(3)$	$+\ (0.02^{+0.05}_{-0.07})$ $-(4.3^{+1.5}_{-1})$
	961.4	$3^- - 4^+$	$-0.095(27)$ $+0.006(32)$	$+0.01(3)$	$+(0.00^{+0.04}_{-0.03})$ $-(9.2^{+4.6}_{-2.4})$
1466.4(2)	1466.4	$2^+ - 0^+$	$+0319(74)$ $-0.041\ (84)$	$E2$	$E2$
	1387.7	$2^+ - 2^+$	$-0.205(36)$ $-0.055(42)$	-($0.73^{+0.12}_{-0.03}$) $-(4.6^{+3.9}_{-1.8})$	$-(0.78^{+0.32}_{-0.17})$ $-(3.8\ ^{+3.8}_{-1.8})$
1476.8(2)	1476.8	$2^+ - 0^+$	$+0.283(50)$ $-0.073(57)$	$E2$	$E2$
	1398.0	$2^+ - 2^+$	$+0.340(47)$ $-0.058(52)$	$+0.15^{+0.12}_{-0.08}$ $+1.5(3)$	$+0.3^{+?}_{-0.18}$ $+1.1^{+0.6}_{-?}$
1600.0(2)	1600.0	$1^- - 0^+$	$-0.155(39)$ $+0.017(45)$	$E1$	E1
	1521.2	$1^- - 2^+$	$+0.011(20)$ $+0.017(23)$	$-0.04(7)$	$-(0.02^{+0.11}_{-0.09})$ $-(3.1^{+1.5}_{0.8})$
1711.2(2)	1632.7	$3^- - 2^+$	$-0.201(47)$ $+0.004(53)$	$+0.03(4)$	$+0.03(7)$ $-(4.7^{+2.3}_{-1.3})$
	1450.9	$3^- - 4^+$	$-0.081(33)$ $+0.030(39)$	$-0.02(3)$	$-0.02(4)$ $-(7.5^{+3}_{-1.7})$

Table (2) : Levels and γ –Transitions of ^{172}Yb [9] Used to Calculate $\rho_2\,(J_i)$

$E_{i\,(keV)}$	$E_{\gamma\,(keV)}$	J_i^π -J_f^π	a_2 a_4[9]	**δ [9]**	$F_2(J_iJ_f\,\delta\,)$ (P.W)	$\rho_2\,(J_i)$ (P.W)	$weighted$ $Average(p.w)$
1155(2)	1155.1	$1^- - 0^+$	$-0.129(54)$ $+0.016(64)$	E1	$+0.70711$	$+0.18243 \pm 0.07637$	-0.20716 ±0.04412
	1076.3	$1^- - 2^+$	$-0.018(29)$ $+0.010(33)$	$+0.01(15)$	$+0.08023$	-0.22435 ± 0.36146	
1600.0(2)	1600.0	$1^- - 0^+$	$-0.155(39)$ $+0.017(45)$	$E1$	$+0.70711$	-0.21920 ± 0.05515	
	1521.2	$1^- - 2^+$	$+0.011(20)$ $+0.017(23)$	$-0.04(7)$	$+0.04683$	-0.23489 $\pm\ 0.43708$	

1117.9(2)	1039.3	$2^+ - 2^+$	+0.183(35) −0.056(40)	−0.02(7) $+2.3^{+0.5}_{-0.3}$	−0.3936 −0.4066	−0.46480 ± 0.08890 −0.45002 ± 0.08607	−0.51549 ±0.02988
	857.4	$2^+ - 4^+$	+0.067(20) −0.030(23)	+0.02(4)	−0.1503	−0.44575 ± 0.13306	
1198.3(2)	1119.6	$2^- - 2^+$	+0.166(50) −0.042(55)	−0.02(10)	−0.3936	−0.42173 ± 0.12703	
1466.4(2)	1466.4	$2^+ - 0^+$	+0319(74) −0.041 (84)	$E2$	−0.59761	−0.53379 ± 0.12383	
	1387.7	$2^+ - 2^+$	−0.205(36) −0.055(42)	$-(0.73^{+0.12}_{-0.03})$ $-(4.6^{+3.9}_{-1.8})$	+0.3548 +0.35764	−0.57768 ± 0.10143 −0.57320 ± 0.10066	
1476.8(2)	1476.8	$2^+ - 0^+$	+0.283(50) −0.073(57)	$E2$	−0.59761	−0.47355 ± 0.08367	
	1398.0	$2^+ - 2^+$	+0.340(47) −0.058(52)	$+0.15^{+0.12}_{-0.08}$ +1.5(3)	−0.58598 −0.60533	−0.58022 ± 0.08021 −0.56168 ± 0.07764	
1608.8(2)	1530.2	$2^+ - 2^+$	+0.052(25) −0.048(28)	$+ 5.7^{+1.8}_{-1.1}$	−0.09671	−0.53769 ± 0.25850	
1172.3(2)	1093.6	$3^+ - 2^+$	−0.089(28) +0.083(35)	$-(7.2^{+1.4}_{-0.9})$	+0.14372	−0.61926 ± 0.19482	−0.65689 ±0.03522
	9119	$3^+ - 4^+$	+0.255(42) +0056(47)	−1.5(4)	−0.40764	−0.62555 ± 0.10303	
1221.8(2)	1143.1	$3^- - 2^+$	−0.193(40) +0.011(48)	+0.02(3)	+0.3082	−0.62603 ± 0.12975	
	961.4	$3^- - 4^+$	−0.095(27) +0.006(32)	+001(3)	+0.15898	−0.59831 ± 0.17005	
1549.8(2)	1471.1	$3^+ - 2^+$	−0.106(42) +0.115(50)	$-(7.0^{+2.0}_{-1.5})$	+0.1513	−0.70050 ± 0.27756	
1700.9(2)	528.4	$3^+ - 3^+$	+0.336(31) −0.047(34)	+0.09(7) +1.1(2)	−0.50502 −0.50280	−0.66532 ± 0.06138 −0.66826 ± 0.06165	
1711.2(2)	1632.7	$3^- - 2^+$	−0.201(47) +0.004(53)	+0.03(4)	+0.2891	−0.69521 ± 0.16256	
	1450.9	$3^- - 4^+$	−0.081(33) +0.030(39)	−0.02(3)	+0.11530	−0.70252 ± 0.28621	
1262.8(2)	1002.5	$4^+ - 4^+$	+0.237(75) −0.0861(83)	−0.17(12) +1.4(3)	−0.27188 −0.29033	−0.87171 ± 0.27586 −0.81631 ± 0.25833	0.05988−0.78285 ±
1286.8	1026.5	$4^+ - 4^+$	+0.383(81) −0.095(92)	$+0.10^{+?}_{-0.17}$ $+0.78^{+0.32}_{-?}$	−0.49820 −0.49804	−0.76877 ± 0.16259 −0.76901 ± 0.16264	
1330.7(2)	1070.4	$4^- - 4^+$	+0.352(98) −0.022(109)	$+0.02^{+0.38}_{-0.16}$	−0.4518	−0.77897 ± 0.21687	
1640.7	377.8	$4^- - 4^+$	+0.358(37) −0.001(40)	+0.03(8)	−0.4583	0.08068−0.78059 ±	
1376.6(3)	204.3	$5^+ - 3^+$	+0.319(85) −0.113(89)	−0.04(9)	−0.37604	−0.84831 ± 0.22604	−0.84831 ± 0.22604

Table (3): Multipole Mixing Ratios of γ −Transitions from Energy levels of ^{172}Yb [9] Calculate by the (CST) Method

$E_{i\,(keV)}$	$E_{\gamma\,(keV)}$	J_i^π -J_f^π	a_2 a_4[9]	δ[9]	δ CST $method$ $(p.w)$
1117.9(2)	1039.3	$2^+ - 2^+$	+0.183(35) −0.056(40)	−0.02(7) $+2.3^{+0.5}_{-0.3}$	−0.05(5) $+ 2.6^{+0.5}_{-0.4}$
	857.4	$2^+ - 4^+$	+0.067(20) −0.030(23)	+0.02(4)	$+0.04^{+0.04}_{-0.039}$ −1.8(2)
1155(2)	1155.1	$1^- - 0^+$	−0.129(54) +0.016(64)	E1	imajenary roots
	1076.3	$1^- - 2^+$	−0.018(29) +0.010(33)	+0.01(15)	$+0.02^{+0.14}_{-0.16}$ $-(3.6^{+4.1}_{-1.4})$
1172.3(2)	1093.6	$3^+ - 2^+$	−0.089(28) +0.083(35)	$-(7.2^{+1.4}_{-0.9})$	$-(7.4^{+1.5}_{-1.0})$ +0.11(2)

	9119	$3^+ - 4^+$	$+0.255(42)$ $+0056(47)$	$-1.5(4)$	$-1.6(3)$ $-0.5(1$
1198.3(2)	1119.6	$2^- - 2^+$	$+0.166(50)$ $-0.042(55)$	$-0.02(10)$	$-(0.08^{+0.07}_{-0.082})$ $+2.8^{+0.8}_{-0.6}$
1221.8(2)	1143.1	$3^- - 2^+$	$-0.193(40)$ $+0.011(48)$	$+0.02(3)$	$+0.03(3)$ $-(4.6^{+0.8}_{-0.6})$
	961.4	$3^- - 4^+$	$-0.095(27)$ $+0.006(32)$	$+001(3)$	$+0.00(3)$ $-(8.8^{+3.0}_{-1.8})$
1262.8(2)	1002.5	$4^+ - 4^+$	$+0.237(75)$ $-0.0861(83)$	$-0.17(12)$ $+1.4(3)$	$-0.18(12)$ $+1.4(3)$
1286.8	1026.5	$4^+ - 4^+$	$+0.383(81)$ $-0.095(92)$	$+0.10^{+?}_{-0.17}$ $+0.78^{+0.32}_{-?}$	$+0.10^{+?}_{-0.18}$ $+0.80^{+0.3}_{-?}$
1330.7(2)	1070.4	$4^- - 4^+$	$+0.352(98)$ $-0.022(109)$	$+0.02^{+0.38}_{-0.16}$	$+0.02^{+?}_{-0.22}$ $+0.9^{+0.4}_{?}$
1376.6(3)	204.3	$5^+ - 3^+$	$+0.319(85)$ $-0.113(89)$	$-0.04(9)$	$-0.04(9)$ Only
1466.4(2)	1466.4	$2^+ - 0^+$	$+0319(74)$ $-0.041\ (84)$	$E2$	imajenary roots
	1387.7	$2^+ - 2^+$	$-0.205(36)$ $-0.055(42)$	$-(0.73^{+0.12}_{-0.03})$ $-(4.6^{+2.0}_{-1.3})$	$-(0.8^{+0.3}_{-0.1})$ $-(3.7^{+1.9}_{-1.4})$
1476.8(2)	1476.8	$2^+ - 0^+$	$+0.283(50)$ $-0.073(57)$	$E2$	majenary roots
	1398.0	$2^+ - 2^+$	$+0.340(47)$ $-0.058(52)$	$+0.15^{+0.12}_{-0.08}$ $+1.5(3)$	$0.23^{+0.15}_{-0.11}$ $+1..3^{+0.4}_{-0.3}$
1549.8(2)	1471.1	$3^+ - 2^+$	$-0.106(42)$ $+0.115(50)$	$7.0^{+2.0}_{-1.5}$	$-(6.8^{+1.9}_{-1.3})$ $+(0.1^{+0.03}_{-0.04})$
1600.0(2)	1600.0	$1^- - 0^+$	$-0.155(39)$ $+0.017(45)$	$E1$	majenary roots
	1521.2	$1^- - 2^+$	$+0.011(20)$ $+0.017(23)$	$-0.04(7)$	$-(0.02^{+0.11}_{-0.1})$ $-(3.1^{+1.7}_{-0.8})$
1608.8(2)	1530.2	$2^+ - 2^+$	$+0.052(25)$ $-0.048(28)$	$+5.7^{+1.8}_{-1.1}$	$+(5.6^{+1.5}_{-1})$ $-(0.25^{+0.03}_{-0.05})$
1640.7	377.8	$4^- - 4^+$	$+0.358(37)$ $-0.001(40)$	$+0.03(8)$	$+0.03^{+0.1}_{-0.09}$ $+0.9(2)$
1700.9(2)	528.4	$3^+ - 3^+$	$+0.336(31)$ $-0.047(34)$	$+0.09(7)$ $+1.1(2)$	$+0.1^{+0.08}_{-0.07}$ $+1.1^{+0.1}_{-0.2}$
1711.2(2)	1632.7	$3^- - 2^+$	$-0.201(47)$ $+0.004(53)$	$+0.03(4)$	$+0.02(4)$ $-(4.4^{+1}_{-0.6})$
	1450.9	$3^- - 4^+$	$-0.081(33)$ $+0.030(39)$	$-0.02(3)$	$-(0.01^{+0.04}_{-0.03})$ $-(7.7^{+3}_{-1.7})$

Table (4): Multipole Mixing Ratios of γ −Transitions from Energy Levels of ^{172}Yb [9] Calculate by the (LSF) Method

$E_{i\,(keV)}$	$E_{\gamma(keV)}$	J_i^π -J_f^π	a_2 a_4[9]	δ[9]	δ $LSF method\ (p.w)$
1117.9(2)	1039.3	$2^+ - 2^+$	$+0.183(35)$ $-0.056(40)$	$-0.02(7)$ $+2.3^{+0.5}_{-0.3}$	-0.04(5) $+2.5^{+0.5}_{-0.3}$
	857.4	$2^+ - 4^+$	$+0.067(20)$ $-0.030(23)$	$+0.02(4)$	$-(1.8^{+0.1}_{-0.2})$ $+(0.04^{+0.03}_{-0.038})$
1155(2)	1155.1	$1^- - 0^+$	$-0.129(54)$ $+0.016(64)$	E1	imajenary roots
	1076.3	$1^- - 2^+$	$-0.018(29)$ $+0.010(33)$	$+0.01(15)$	$+0.02^{+0.18}_{-0.16}$ $-(3.6^{+3.9}_{-1.4})$

1172.3(2)	1093.6	$3^+ - 2^+$	$-0.089(28)$ $+0.083(35)$	$-(7.2^{+1.4}_{-0.9})$	$-(7.5^{+1.5}_{-1.0})$ $+0.11(2)$
	9119	$3^+ - 4^+$	$+0.255(42)$ $+0056(47)$	$-1.5(4)$	$-1.6(3)$ $-0.5(1)$
1198.3(2)	1119.6	$2^- - 2^+$	$+0.166(50)$ $-0.042(55)$	$-0.02(10)$	$-(0.07^{+0.08}_{-0.078})$ $2.7^{+0.9}_{-0.5}$
1221.8(2)	1143.1	$3^- - 2^+$	$-0.193(40)$ $+0.011(48)$	$+0.02(3)$	$0.03(3)$ $-(4.7^{+0.8}_{-0.7})$
	961.4	$3^- - 4^+$	$-0.095(27)$ $+0.006(32)$	$+001(3)$	$0.00(3)$ $-(8.6^{+2.7}_{-1.7})$
1262.8(2)	1002.5	$4^+ - 4^+$	$+0.237(75)$ $-0.0861(83)$	$-0.17(12)$ $+1.4(3)$	$-0.17(11)$ $+(1.3^{+0.4}_{-0.2})$
1286.8	1026.5	$4^+ - 4^+$	$+0.383(81)$ $-0.095(92)$	$+0.10^{+?}_{-0.17}$ $+0.78^{+0.32}_{-?}$	$+0.10^{+?}_{-0.17}$ $+0.78^{+0.32}_{-?}$
1330.7(2)	1070.4	$4^- - 4^+$	$+0.352(98)$ $-0.022(109)$	$+0.02^{+0.38}_{-0.16}$	$+0.03^{+?}_{-0.17}$ $+0.9^{+0.4}_{-?}$
1376.6(3)	204.3	$5^+ - 3^+$	$+0.319(85)$ $-0.113(89)$	$-0.04(9)$	$-0.04(9)$ Only
1466.4(2)	1466.4	$2^+ - 0^+$	$+0319(74)$ $-0.041(84)$	$E2$	imajenary roots
	1387.7	$2^+ - 2^+$	$-0.205(36)$ $-0.055(42)$	$-(0.73^{+0.12}_{-0.03})$ $-(4.6^{+2.0}_{-1.3})$	$-(0.8^{+0.2}_{-0.1})$ $-(3.6^{+1.6}_{-1.1})$
1476.8(2)	1476.8	$2^+ - 0^+$	$+0.283(50)$ $-0.073(57)$	$E2$	imajenary roots
	1398.0	$2^+ - 2^+$	$+0.340(47)$ $-0.058(52)$	$+0.15^{+0.12}_{-0.08}$ $+1.5(3)$	$+0.3^{+0.11}_{-0.15}$ $+1.3(3)$
1549.8(2)	1471.1	$3^+ - 2^+$	$-0.106(42)$ $+0.115(50)$	$7.0^{+2.0}_{-1.5}$	$-(6.9^{+1.9}_{-1.4})$ $+0.1(30)$
1600.0(2)	1600.0	$1^- - 0^+$	$-0.155(39)$ $+0.017(45)$	$E1$	imajenary roots
	1521.2	$1^- - 2^+$	$+0.011(20)$ $+0.017(23)$	$-0.04(7)$	$-(0.02^{+0.11}_{-0.1})$ $-(3.1^{+1.6}_{-0.8})$
1608.8(2)	1530.2	$2^+ - 2^+$	$+0.052(25)$ $-0.048(28)$	$+5.7^{+1.8}_{-1.1}$	$+(5.5^{+1.5}_{-1})$ $-0.25(4)$
1640.7(3)	377.8	$4^- - 4^+$	$+0.358(37)$ $-0.001(40)$	$+0.03(8)$	$+0.04^{+0.06}_{-0.07}$ $+0.9^{+0.1}_{-0.2}$
1700.9(2)	528.4	$3^+ - 3^+$	$+0.336(31)$ $-0.047(34)$	$+0.09(7)$ $+1.1(2)$	$+0.08(6)$ $+1.1^{+0.2}_{-0.1}$
1711.2(2)	1632.7	$3^- - 2^+$	$-0.201(47)$ $+0.004(53)$	$+0.03(4)$	$+(0.03^{+0.03}_{-0.02})$ $-(4.5^{+0.9}_{-0.7})$
	1450.9	$3^- - 4^+$	$-0.081(33)$ $+0.030(39)$	$-0.02(3)$	$-(0.02^{+0.03}_{-0..04})$ $-(7.6^{+2.7}_{-1.6})$

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